





Hough - Transform-based Interpolation Scheme for Generating Dense Spatial Maps of Air Pollutants from Sparse Sensing

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Air Pollution: Public Health and the Effects on the Environment

- Significant risk factor for multiple health situations including eye irritation, breathing difficulties, lung cancer, heart diseases and respiratory infections.
- Cause many negative effects on the environment: decreased visibility, acid rain, global warming, climate change, water quality deterioration and ecosystems destruction.
- Importance of assessing air-quality



Air Quality Monitoring (AQM) Station and Sensors

- Today, many air-pollution studies based on data acquired from AQM.
- Provides continuous and accurate measurements.
- Expansive to build and operate
- Sited mainly near 'hot- spots'- where the pollution level might be high or near places of interest
- Sensors many benefits same problem



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Information obtained from AQM has to be generalized with mathematical methods



Spatial Coverage - Interpolation Schemes



paulbourke.net

- Interpolation is a mathematical method of constructing continuous function within the range of measured points.
- Environmental interpolation:
 - Deterministic: influence diminishes with distance (IDW, Nearest Neighbor)
 - Geostatistical: autocorrelation, asses the statistical relationships among the measured points (Kriging)
- ► Focus on: IDW and Ordinary Kriging





What's the problem with IDW

IDW: Equation Analysis



HTBI: Hough-Transform-based Interpolation Scheme



Hough Transform

Cartesian space

Parametric space

quora.com.



Proof of Concept – Source Detection



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HTBI Notation

Let $\{S\}$ be a set of sources of specific pollutant, with emission rates $\{Q\}$.

Let A be a continues pollution signal generated by $\{S\}$, defined over a geographical area Ω .

{*S*} are located at $\vec{\gamma} \in \Omega$.

Let $\{a\}$ be a finite set of samples of signal A, taken in locations $\{\omega\} \subset \Omega$.

Interpolation aims at estimating A over the entire space Ω , based on the set of samples $\{a\}$

We assume A is complies with a uniform model over entire Ω .

Stage #1 Source Detection by HTBI

Each sample a_i is a weighted combination of the contributions from all the sources emissions, \vec{Q} , under some dispersion model, M. Hence a_i is given by:

$$a_i = \vec{M}_i \cdot \vec{Q}^T$$

All sensors' measurements can be represented by the following matrices multiplication:

$$\vec{a} = [M] \cdot \vec{Q}^T$$

Given [M], we assume that there exists matrix E, which satisfies :

$$\vec{Q} = [E] \, \vec{a}^T$$

Source Detection by HTBI cont.

- Focus on single source detection
- ► We divide the feature space Ω into *N* disjoint catchments, $C_n \subseteq \Omega$
- For each of the catchments, an estimated emission rate \hat{Q}_n^i is calculated, based on accepted measurements from single sample a_i :

$$\hat{Q}^{i}{}_{n} = e \cdot a_{i}$$

where e is a single row of E
A full hypothetical emission rate:

$$\vec{\hat{Q}}_n = [E] \cdot \vec{a}^T$$

Source Detection by HTBI cont.

Let σ_n be the standard deviation (STD) of $\vec{\hat{Q}}_n$:

$$\sigma_n = STD(\vec{\hat{Q}}_n)$$

The catchment with the lowest σ is the approximate location of *S*, i.e. γ :

$$\gamma = MIN(\sigma_n)$$

Once the source location, γ, is obtained, the emission rate of S is estimated by the average of the catchment's estimates:

$$\widehat{Q}_{\gamma} = \overline{\widehat{Q}}_{\gamma}$$

Dispersion Models



$$a_i = Q \cdot e^{-\lambda |r|}$$

(Buhmann, 2003)

Gaussian Plume Dispersion (GPD) model :

$$a_{i} = \frac{Q}{2\pi\sigma_{y}\sigma_{z}\bar{u}} \exp(-\frac{y^{2}}{2\sigma_{y}^{2}}) [\exp(-\frac{(z-H)^{2}}{2\sigma_{z}^{2}}) + \exp(-\frac{(z+H)^{2}}{2\sigma_{z}^{2}})]$$

(Ermak, 1977)

Stage #2: Creation of Dense Spatial Pollution Map

• We can now estimate the entire Ω :

$$C_n = \vec{M} \cdot \hat{Q}_{\gamma}$$

Computational Simulation

- Geographical area Ω with a size of 20km²
- \blacktriangleright Q = 8 ton/hour
- ► For Gaussian plume model:
 - Effective stack-height: 120m
 - ► Wind:
 - Speed = 4 m/sec
 - Direction = 285°
 - Stability class: stable



Additive white Gaussian noise with SNR of 10%

Computational Simulation -

Sensor #	Radial (µg/m ³)	GPD (μg/m ³)
(1)	0.180	0
(2)	2.586	0
(3)	3.35*e-10	6.59*e-30
(4)	1.60*e-16	282.5981
(5)	1.65*e-20	6.20*e-17
(6)	1.23*e-30	1.1097



Results #1: Radial Dispersion Model





Kriging



IDW

Results #2: Gaussian Dispersion Model



Uncertainty of the Models

- Additive white Gaussian noise on both ambient concentrations, wind speed and direction with differential SNR
- Radial Model: shows stability even with SNR > 50% (SNR of 3dB)
- Gaussian Model: Shows dependency on the catchments size:
 - For cell size of 40m², our algorithm showed stability up to 10% SNR.
 - For cell size of 20m², the HTBI showed higher
 - sensitivity to noise only up to 5% SNR (13 dB).



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Future Work

- Future work, carried out these days, is focusing on the implementation of the method on a real-world problem.
- Preliminary results obtained, reveal the high potential inherent in our method
- Much work to be done to adapt HTBI to real conditions
- In particularly, the existing dispersion models are based on several preliminary assumptions that are not always consistent with the actual situation

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